TWO-DIMENSIONAL STEADY CONDUCTION ANALYSIS

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Study objective

This case study shows the application of SIMple® Thermal to solve a heat transfer problem proposed in (Eastop & McConkey, 1993). The problem is a theoretical example for the two-dimensional steady conduction subsection in the book heat transfer chapter. The solution of such problems requires numerical methods for meshing the physical system and obtaining values for temperature and heat flow along the mesh.

The problem is: a long duct of square section 0.5 m x 0.5 m is buried in deep soil with one of its sides parallel to the surface of the soil as shown in Figure 1; the center-line of the duct is at a depth of 1.25 m. The surface of the soil is at an equilibrium temperature of 0°C, and at a soil depth of 2.5m it may be assumed that the uniform equilibrium temperature is -10°C across the horizontal cross-section. The temperature of each side of the duct is 50°C and it may be assumed that at a vertical cross-section a horizontal distance of 2.25 m from the duct center-line, the heat transfer vertically downwards from the surface is simple one-dimensional conduction. Taking a square mesh of side 0.5 m, use a numerical method to obtain an approximation for the temperatures within the soil to the nearest degree, and estimate the heat loss per meter length of duct. The thermal conductivity of the soil is 1 W/m K.

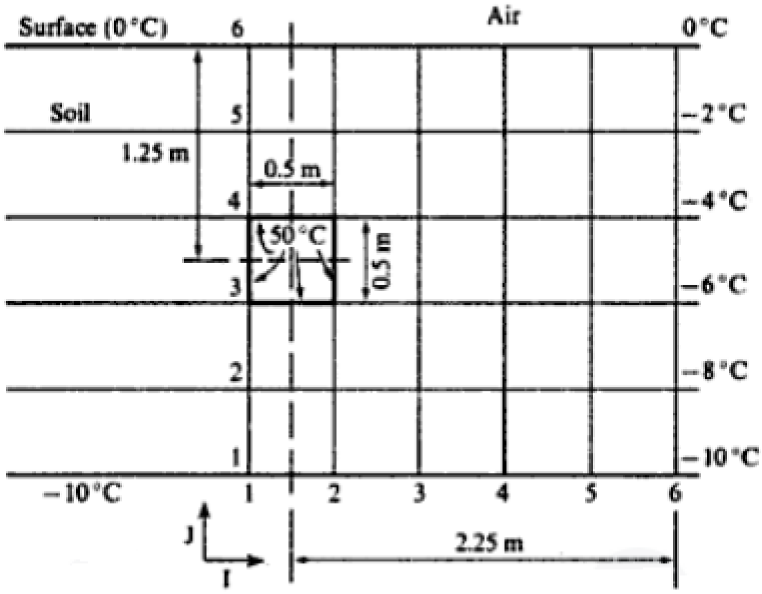

Figure 1 - Duct buried in soil for the example, taken from (Eastop & McConkey, 1993).

Methodology

The problem states that a square mesh should be taken to obtain a numerical solution for the temperatures within the soil. This mesh can be easily drawn with the blocks from the SIMple® Thermal library. More specifically, the blocks listed in Table 1 can be used.

Table 1 - Blocks from SIMple® Thermal used for creating the mesh.

Heat Transfer > **HeatCapacitor** 	This block can be used to represent each internal node within the mesh. Nodes are basically points where the temperature must be calculated. Although in this case study we are only interested in steady-state conduction, the fact that this block is designed for modeling thermal inertia would also allow a transient conduction analysis.
Heat Transfer > **Conduction** 	This block can be used to represent the conduction heat transfer phenomenon in a single direction. As this study involves a 2-D heat conduction case, the diagram must have blocks for conduction in both coordinates I and J.
Boundary Conditions > **HeatSource** 	This block can set boundary values for both heat flow and temperature. In this case study, it should be used for setting the temperature value wherever it is known.
Sensors > **HeatFlowSensor** 	This block must be used for exporting heat flow values along the mesh. A sensor must be placed alongside every conduction block next to the sources representing the duct, so that the overall heat loss per meter length of duct can be calculated.

Figure 2 shows the block diagram created with those blocks. Some native blocks from Altair™ Activate are also present in the diagram: the Scope block is used alongside every HeatCapacitor instance, so that the temperature value for each mesh node can be plotted in a chart during the model convergence, and the Sum and Gain blocks are used for calculating the total heat loss of the duct. The total heat loss is equal to the sum of the heat flows calculated in the conduction blocks right next to the duct boundary conditions multiplied by two, as the problem is symmetrical and the mesh only represents one of its sides.

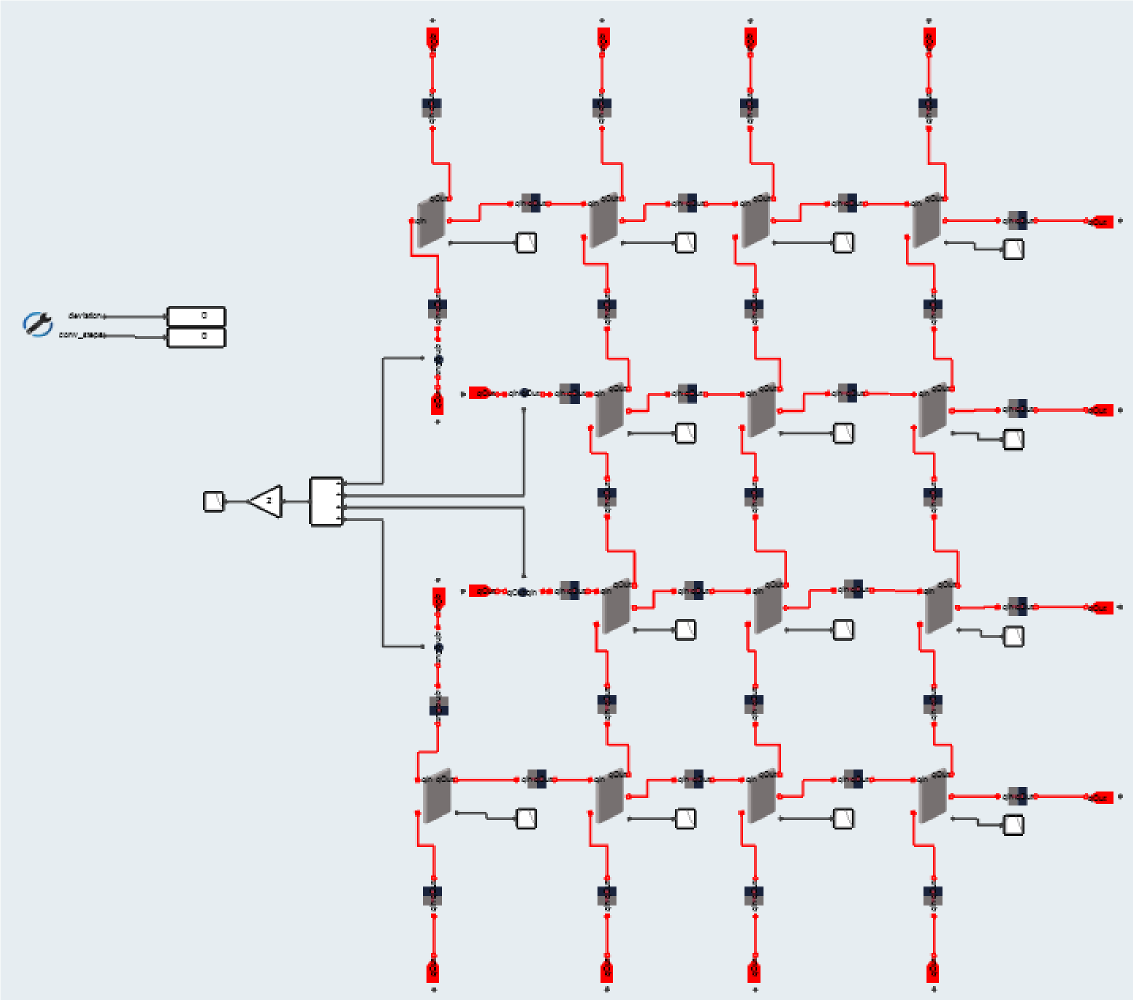

Figure 2 - Model assembled with blocks from SIMple® Thermal.

Each HeatSource block is set up with its corresponding temperature value. The temperature is set a 0°C for the sources representing the surface, -10°C for the ones representing the soil at a depth of 2.5 m, 50°C for the ones associated with the duct boundary conditions and -2°C, -4°C, -6°C and -8°C for the ones at a horizontal distance of 2.25 m from the duct center-line.

As soil properties are considered constant at any height or horizontal position, the 37 conduction blocks are set up with the exact same parameters: 1 W/m K for thermal conductivity, 0.5 m for length and 0.25 m^2 for cross-section area. This cross-section area considers that each element is one meter long in the duct axial direction, because the heat loss must be calculated per meter length of duct. Finally, the heat capacitor blocks are configured with temperature initial conditions, which in a steady-state test can be interpreted as the initial guesses for the problem's solution. An initial guess of -5°C is given for all nodes.

Results

Running the simulation in Altair™ Activate, the results shown in Figure 3 are obtained.

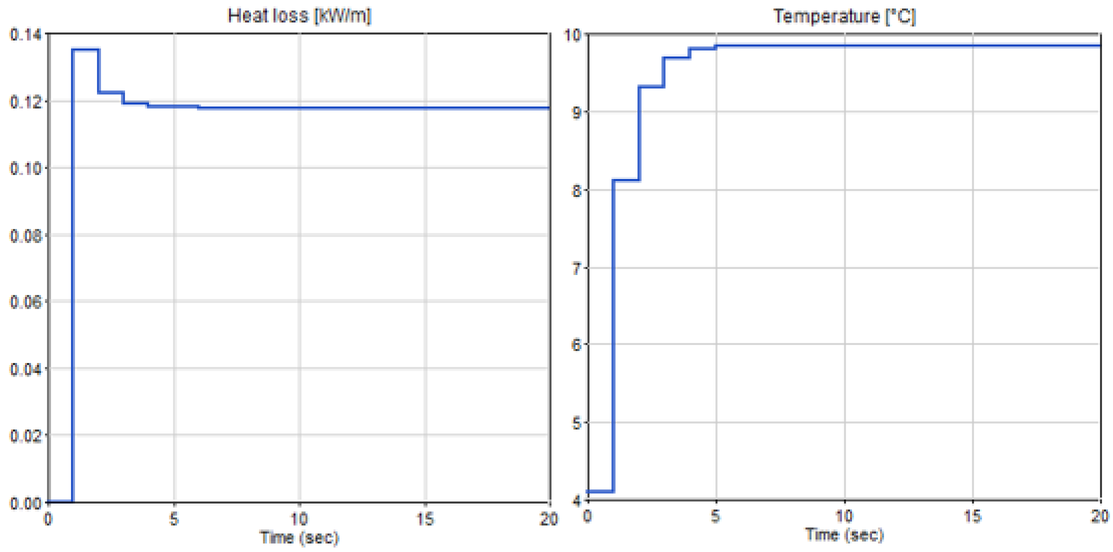

Figure 3 - Results for the case study simulation.

The first chart in Figure 3 has the values for the duct heat loss during the simulation convergence, while the second one is for the temperature at one of the internal nodes. It can be seen that the model converges to a solution within approximately six steps. Heat loss converges to a value of 117,6 W/m, which is very close to the value of 118 W/m predicted at (Eastop & McConkey, 1993), which is not an exact solution, but the result of a single iteration.

Table 2 has the results for all the system's internal temperatures. A first glance at the results reveals that the temperature is higher at the nodes close to the duct, which is according to the expected.

Table 2 - Numerical results for the temperature within the soil.

Temperature (°C)		Horizontal distance from the duct center-line (m)					
(values in bold are calculated by the model, others are boundary conditions)		-0.25	0.25	0.75	1.25	1.75	2.25
Depth (m)	0	0	0	0	0	0	0
	0.5	**20.88**	**20.88**	**12.63**	**5.98**	**1.44**	-2
	1.0	50	50	**23.66**	**9.85**	**1.77**	-4

	1.5	50	50	**22.15**	**7.98**	**-0.19**	-6
	2.0	**15.66**	**15.66**	**6.98**	**0.11**	**-4.52**	-8
	2.5	-10	-10	-10	-10	-10	-10

Theory provided at (Eastop & McConkey, 1993) describes that the finite-difference solution of the two-dimensional steady heat conduction equation (also known as the Laplace equation) implies that, for a grid with square elements, temperature at each internal point equals the average of its four neighbor points' temperatures. This can be used to validate the simulation results. For the highlighted values at Table 2, one can quickly check that:

$$23.66 = \frac{12.63 + 9.85 + 22.15 + 50}{4}$$

The same verification can be done for all the calculated values from Table 2. Therefore, the results are physically consistent, because they are indeed a solution for the Laplace equation.

References

Eastop, T., & McConkey, A. (1993). *Applied Thermodynamics For Engineering Technologists* (5th ed.). Longman Scientific & Technical.